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The Soul Theorem

Mathieu Wydra

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Manifolds		

Geodesic 00 The Theorems

Outline



2 Geodesics



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A brief introduction to Riemannian geometry

Smooth manifolds

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A brief introduction to Riemannian geometry

- Smooth manifolds
- Tangent Spaces

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The Theorems

A brief introduction to Riemannian geometry

- Smooth manifolds
- Tangent Spaces
- Vector Fields

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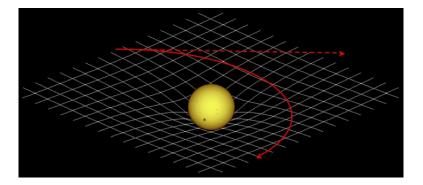
A brief introduction to Riemannian geometry

- Smooth manifolds
- Tangent Spaces
- Vector Fields
- Connections

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Geodesics ●0 The Theorems

Geodesics

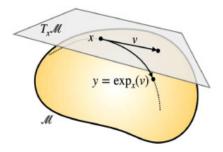


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Theorem (Cohn-Vossen)

A complete non-compact manifold of dimension 2 with everywhere non-negative (Riemannian) curvature is either diffeomorphic to \mathbb{R}^2 or is flat.

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Theorem (Cohn-Vossen)

A complete non-compact manifold of dimension 2 with everywhere non-negative (Riemannian) curvature is either diffeomorphic to \mathbb{R}^2 or is flat.

Theorem (Soul Theorem)

Let M be a complete non-compact manifold with non-negative sectional curvature. Then M contains a compact, totally geodesic and totally convex submanifold S whose normal bundle is diffeomorphic to M.

Theorem (Soul Conjecture)

Let M be a complete, connected and non-compact manifold with sectional curvature $K \ge 0$ and there exists a point in M where the sectional curvature is strictly positive. Then the soul of M is a point. Furthermore, M is diffeomorphic to \mathbb{R}^n .

The Theorems

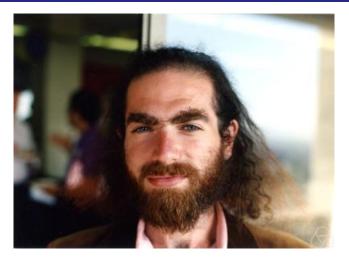


Figure: Grigori Perelman

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Theorem (Perelman)

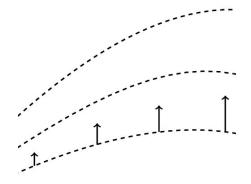
Let M be a complete non-compact manifold of non-negative sectional curvature, let S be the soul of M and let $P : M \rightarrow S$ be a distance non-increasing retraction. Then

1 For all $x \in S$, $\nu \in SN(S)$, we have

$$P(\exp_x(t\nu) = x, \quad \forall t \ge 0$$

2 For any geodesic $\gamma \subset S$ and vector field $\nu \in \Gamma(SN(S))$ parallel along γ the 'horizontal curves' γ_t ; $\gamma_t(u) = \exp_{\gamma(u)}(t\nu)$ are geodesics, filling a totally geodesic flat strip $(t \ge 0)$. Moreover, if $\gamma[u_0, u_1]$ is minimising, then all $\gamma_t[u_0, u_1]$ are also minimising.

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